A Coupled Meshfree/Meshbased Method for Geometrically Complex FSI Problems

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Motivation

- In geometrically complex FSI problems, a mesh may be difficult to be maintained $\Rightarrow$ meshbased methods like the FEM may fail.
- For example: large geometry deformations, moving and rotating objects
Motivation

- Use meshfree methods (MMs): no mesh needed, but more time-consuming
- Coupling: Use MM only where mesh makes problems, otherwise employ FEM.

Fluid: Coupled MM/FEM
Structure: Standard FEM
Meshfree Method

- Meshfree method for the fluid part:
  - Approx. incompressible Navier-Stokes equations.
  - Moving least-squares (MLS) functions in a Galerkin setting.
  - Nodes are fixed: Eulerian or ALE formulation,

Closely related to the **element-free Galerkin (EFG)** method.

- For the success of this method,
  - stabilization and
  - coupling with meshbased methods is needed.
Meshfree Method

**Element-free Galerkin method (EFG)**
- MLS functions in Galerkin setting.

**Moving Least Squares (MLS)**
- Discretize the domain by a set of nodes $x_i$
- Define local weighting functions $w(x - x_i)$ around each node
Meshfree Methods

- Local approximation around an arbitrary point
  
  \[ u^h(x) = p^T(x)a(x) \]
  
  - \( p(x) \): complete basis vector, e.g. \([1 \ x \ y]\)
  - \( a(x) \): unknown coefficients of the approximation

- Minimize a weighted error functional
  
  \[ J(a) = \sum_{i=1}^{N} w(x-x_i) \left[ p^T(x)a(x)-u(x_i) \right]^2 \Rightarrow Ma = Bu \]

- For the approximation follows:
  
  \[ u^h(x) = p^T M^{-1} B u \]

  \( N^T(x) \): meshfree MLS functions
Meshfree Methods

MLS functions

Supports in MMs are adjustable

lin. FEM functions

Supports in FEM are pre-defined by mesh
Stabilization

- Incompressible Navier-Stokes equations:
  - Momentum equation: \( \rho \left( u_t + u \cdot \nabla u - f \right) - \nabla \cdot \sigma = 0 \)
  - Continuity equation: \( \nabla \cdot u = 0 \)
  - Newtonian fluid: \( \sigma(u, p) = -pI + 2\mu \varepsilon(u), \quad \varepsilon(u) = \frac{1}{2} \left( \nabla u + (\nabla u)^T \right) \)

- Stabilization is necessary for
  - Advections terms: NS-eqts. in Eulerian or ALE formulation
  - “Equal-order-interpolation“: incompr. NS-eqts. with the same shape functions for velocity and pressure.

- Stabilization is realized by SUPG/PSPG
Stabilization

- SUPG/PSPG-stabilized weak form (Petrov-Galerkin):

\[
\int \tilde{w} \rho (\tilde{u}_t + \tilde{u} \cdot \nabla \tilde{u} - \tilde{f}) \, d\Omega + \int \varepsilon (\tilde{w}) : \tilde{\sigma} (\tilde{u}, \tilde{p}) \, d\Omega + \\
\int \tilde{q} \nabla \cdot \tilde{u} \, d\Omega \\
+ \sum_{i=1}^{n_{\text{el}}} \int_{\tau} \left( \tilde{u} \cdot \nabla \tilde{w} + \frac{1}{\rho} \nabla \tilde{q} \right) \cdot \left[ \rho (\tilde{u}_t + \tilde{u} \cdot \nabla \tilde{u} - \tilde{f}) - \nabla \cdot \tilde{\sigma} (\tilde{u}, \tilde{p}) \right] \, d\Omega \\
= \oint \tilde{w} \tilde{h} \, d\Gamma
\]
Stabilization

- The structure of the stabilization method may be applied to meshbased *and* meshfree methods.
- The stabilization parameter $\tau$ requires special attention:
  - MLS functions are not interpolating $\Rightarrow$ different character
  - Standard formulas are only valid for *small* supports.
- SUPG/PSPG-stabilization turned out to be slightly less diffusive than GLS.
Coupling

- Our aim is a fluid solver for geometrically complex situations.
  MMs shall be used where a mesh causes problems, in the rest of the domain the efficient meshbased FEM is used.
- The coupling is realized on shape function level.
Coupling

- Coupling approach of Huerta et al.: Coupling with modified MLS technique, which considers the FEM influence in the transition area.

\[
N_i^{\text{MLS}}(x) = \left[ p^T(x) - \sum_{j \in I^{\text{FEM}}} N_j^{\text{FEM}}(x) p^T(x_j) \right]
\]

\[
M^{-1}(x) w(x - x_i) p(x_i)
\]

- Modifications are required in order to obtain coupled shape functions that may be stabilized reliably.
Coupling

- Original approach of Huerta et al.

- Modification: Add MLS nodes along $\Omega^{\text{FEM}} \cap \Omega^*$ and superimpose shape functions there.

\[
\rho_i > 2.0 \Delta x
\]

\[
\rho_i > 1.0 \Delta x
\]
Coupling

original:

modified:

- Superimpose FEM and MLS nodes along $\Omega^{\text{FEM}} \setminus \Omega^*$ ⇒ smaller supports ⇒ reliable stabilization
- The resulting stabilized and coupled flow solver has been validated by a number of test cases.
Fluid-Structure Interaction

- Situation:

- Strongly coupled, partitioned strategy:

Fluid: coupled MLS/FEM

Structure: FEM

Mesh movement

Fluid domain $\Omega_F$

Structure domain $\Omega_S$

Traction along $\Gamma_C$

Displacement of $\Gamma_C$
Numerical Results

- Vortex excited beam

\[ u = 51.3 \]
\[ v = 0 \]
\[ \text{flap} \]
\[ \text{Re} = 333 \]

![Diagram](image)

- Vertical displacement at flap tip

\[ d_y \]

- Meshfree and FEM solutions

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Numerical Results

vorticity

a)  b)
Numerical Results

- Rotating object

\[ u_{\text{max}} = 1.5 \]
\[ v = 0 \]

\[ u = 0, v = 0 \]

\[ \text{Re} = 100 \]
Numerical Results
Numerical Results
Numerical Results

(a) deformation and stress

(b) deformation at rotor tip

\[ \alpha = 45^\circ \]
Numerical Results

- Moving flap

\[ u_{\text{max}} = 1.5 \quad v = 0 \]

\[ \alpha \]

\[ \Omega^{\text{FEH}} \]

\[ \Omega^{\text{MLS}} \]

\[ \text{Re} = 100 \]
Numerical Results

- The flap is considered as a discontinuity.
- “Visibility criterion” is used for the construction of the MLS-functions.
Numerical Results
Summary and Outlook

Summary

- Meshfree Method: Eulerian (ALE) formulation, MLS functions in a Galerkin setting ⇒ EFG
- SUPG/PSPG-Stabilization
- Modified coupling approach of Huerta
- Numerical results for geometrically complex FSI problems

Outlook

- More test cases
- Coupled shape functions of higher order
- Extension to three-dimensional problems