Simulation of Swimming Objects with the XFEM

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Introduction

- Classical FSI: Structure is completely immersed in a fluid.
- Example: Square cylinder with flexible flap.

This is an algorithmic three-field problem:

Fluid (ALE) → Structure → Mesh
Introduction

- Floating objects: The structure penetrates the free surface of the fluid, i.e. it is *not* fully immersed.
- Special case: The structure is not even in contact with the fluid in the initial situation.
Introduction

- Reformulation as a two-fluid flow problem with a fully immersed structure: Artificial fluid ("air") has negligible density and viscosity.
Interfaces

- There is a fluid-fluid and fluid-structure interface.

- The fluid-structure interface is meshed *explicitly*, the fluid and structure meshes conform (interface tracking).

- The fluid-fluid interface is meshed *implicitly* by means of the level-set method (interface capturing).
Interfaces

Example:

- At the fluid-fluid interface, the large difference in the viscosity & density of the two fluids leads to severe kinks in the velocity & pressure fields.
- These kinks are inside elements.
The XFEM allows the approximation of inner-element discontinuities (here: kinks) with optimal accuracy.

This is achieved by a local enrichment of the approximation space.

\[ u^h (x) = \sum_i N_i (x) u_i + \sum_{I^*} M_i (x) a_i + \ldots, \]

with

\[ M_i (x) = N_i (x) \cdot \psi (x), \]

where \( \psi (x) \) is the enrichment function.
XFEM for two-fluid flows

- $I^*$ and $\psi(x)$ define a particular realization of the XFEM.
- $I^*$ is the set of nodes of all cut elements.

**Interface definition by $\phi(x)$**

$\psi(x)$ depends on the level-set function for the definition of the interface:

- **Weak discontinuities:** $\psi(x) = \text{abs}(\phi(x))$
- **Strong discontinuities:** $\psi(x) = \text{sign}(\phi(x))$
**XFEM for two-fluid flows**

- Governing equations for the flow are the incompressible Navier-Stokes equations in velocity-pressure formulation.

- Different enrichment variants have been realized:
  - Variant 1: Velocity and pressure fields are abs-enriched.
  - Variant 2: Velocity fields are not enriched, pressure field is abs-enriched.
  - Variant 3: Velocity fields are not enriched, pressure field is sign-enriched.

- Two important results:
  - The enrichment of the pressure field is crucial, velocity fields may not be enriched.
  - When surface tension effects are neglected, the pressure field should be abs-enriched, otherwise, sign-enriched.
**XFEM for two-fluid flows**

**Time integration**
- For efficiency, time-stepping is used, but special considerations are required for XFEM.  
  [Fries & Zilian, IJNME, 2009]
- Example:
  - Consider a stationary mesh.
  - The moving fluid-fluid interface is described by a time-dependent level-set function.
  - The enrichment functions depend directly on the level-set function.
  - Consequently, in contrast to classical FEM, there are time-dependent shape functions even for stationary meshes.
**XFEM for two-fluid flows**

**Quadrature**

- In time-stepping schemes, at least two time-levels are considered: \( t_n \) and \( t_{n+1} \).
- There are two different corresponding approximations, \( u_n^h \) and \( u_{n+1}^h \), which are discontinuous at the interfaces described by \( \phi_n \) and \( \phi_{n+1} \).

![Integration Point](image)

- Decomposition w.r.t. to both time-levels.
**Blending elements**

- Recall: Nodes in *local* parts of the domain are enriched.
- Elements are (i) completely enriched, (ii) partly enriched, or (iii) not enriched.

In blending elements, parasitic terms are introduced for the abs-enrichment, can be overcome by the Corrected XFEM.

[Fries, IJNME, 2007]
Coupling

- We use a strongly coupled, partitioned strategy for the coupling of the *four* fields:

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Fluid 1+2 (ALE)

Interface

Structure

Mesh
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Numerical results

- Test-case: Object falling onto free surface.

\[ \begin{align*}
\rho_2 &= 1 \\
\mu_2 &= 0.01 \\
r &= 0.1 \\
\rho_1 &= 1000 \\
\mu_1 &= 0.1
\end{align*} \]

\[ L = 1.0 \quad H = 1.75 \]
Conclusions

- For the simulation of floating objects, adding an auxiliary field recovers the classical situation where the structure is completely immersed in the fluid(s).
- The problem is shifted to the accurate solution of a two-fluid problem with interface capturing.
- The XFEM is highly suitable for such an application.
- The enrichment of the pressure field is crucial.
- The XFEM can be used successfully for the simulation of floating bodies.


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References

The XFEM: Some facts

What is different in an XFEM-code (compared to FEM)?

- Enrichment functions are evaluated.
- Special integration rules are used for cut elements.
- The number of DOFs per node varies (element matrices, overall system of equations).
- Possibly the post-processing.

⇒

Special considerations may be needed for...

- ...applying boundary/interface conditions. [Zilian, Fries, IJNME, 2009]
- ...choosing appropriate time-integration schemes. [Fries, Zilian, IJNME, 2009]
- ...bad condition numbers in some situations.
Properties of XFEM

Important properties of the standard XFEM:

- The enrichment is local.
- The enrichment is added extrinsically.
- There are more unknowns than in the standard FEM.
- The numerical integration has to consider the discontinuity in the functions.
- The approximated function $u^h(x)$ has to be assembled from all terms in the approximation.

\[ u^h(x) = \sum_i N_i(x) u_i + \sum_{I^*} M_i(x) a_i + \ldots, \]