The XFEM using Crack Tip Enrichment with Large Support for Curved Cracks

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Outline

1. Motivation
2. New Alternative
3. Other Alternatives
4. Studies
5. Conclusions
eXtended Finite Element Formulation

\[ u(x, y) = \sum_{i \in I} N_i(x, y)u_i + \sum_{j \in I^1} N_j^+(x, y) \cdot H(x, y)a_j + \sum_{k \in I^2} N_k^+(x, y) \cdot \left( \sum_{m=1}^{4} B^m b_k^m \right) \]

Continuous

Discontinuous
eXtended Finite Element Formulation

\[ u(x, y) = \sum_{i \in I} N_i(x, y) u_i + \sum_{j \in I^+} N_j^+(x, y) \cdot H(x, y) a_j + \sum_{k \in I^+} N_k^+(x, y) \cdot \left( \sum_{m=1}^{4} B^m b_k^m \right) \]

Continuous

Discontinuous

\[
\Gamma
\]

\[
\Gamma
\]
In XFEM, Optimal Convergence Rates with Fixed Radius for Branch Enrichments are achieved. [LABORDE ET AL.]
Coordinate Systems

For Curved Cracks/Crack Propagation, Different Coordinate Systems to evaluate the SIFs and/or Enrichments exist:

Alternative 0

- Easy to evaluate by Crack Tip Information only.
- Discontinuity follows a Straight Path.
- Not Suitable for “Large” Radius Enrichment.
Coordinate Systems

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- Easy to evaluate by Crack Tip Information only.
- Discontinuity follows a Straight Path.
- Not Suitable for “Large” Radius Enrichment.

Alternative 1
- Discontinuity follows Curved Path.
- Bases $e_1$ and $e_2$.
- Drawbacks.
Drawbacks of Alternative 1

- Two Level Set Functions: $\phi$ and $\gamma$.
- $\forall p \in \Omega$: Two Signed Values and Bases.

- Bases are no longer Orthogonal.
- Inconvenient values away from the Tip.
Drawbacks of Alternative 1

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Motivation

New Alternative

Other Alternatives

Studies
Signed Distance
Derivatives of Level Sets
Radius Enrichment
Stress Intensity Factors

Conclusions

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Alternative 2

- Geometrical Reconstruction.
- Split the Domain into Triangles.
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Alternative 2

- $\forall p \in$ Triangle.
- Interpolate Limiter Level Sets $\gamma$.
- Interpolate Signed Distance for point $p$.
- Find Signed Distance $\phi$. 

![Diagram showing level sets and signed distances](image-url)
Alternative 2

- $\forall p \in \text{Triangle}$.
- Interpolate Limiter Level Sets $\gamma$.
- Interpolate Signed Distance for point $p$.
- Find Signed Distance $\phi$.
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- Interpolate Limiter Level Sets $\gamma$. 
- Interpolate Signed Distance for point $p$. 
- Find Signed Distance $\phi$. 

$\gamma = -da$
$\gamma = 7da/8$
$\gamma = -6da/8$
$\gamma = -5da/8$
$\gamma = -da/2$
$\gamma = -3da/8$
$\gamma = -da/4$
$\gamma = -da/8$
$\gamma = 0$
Alternative 2

- \( \forall p \in \text{Triangle}. \)
- Interpolate Limiter Level Sets \( \gamma \).
- Interpolate Signed Distance for point \( p \).
- Find Signed Distance \( \phi \).
Alternative 2

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\[
\begin{align*}
\gamma &= \text{da} \\
\gamma &= 7\text{da}/8 \\
\gamma &= -6\text{da}/8 \\
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\gamma &= -3\text{da}/8 \\
\gamma &= -\text{da}/4 \\
\gamma &= -\text{da}/8 \\
\gamma &= 0
\end{align*}
\]

\[
\begin{align*}
\Phi &= 0 \\
\Phi &= 3
\end{align*}
\]
Alternative 2

- Convenient Values $\gamma$.
- Bases are almost Orthogonal.
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\[\Phi = 2.7, \quad \gamma = -2.65 \text{da}\]
Other Alternatives

Alternative 2– a

- Applicable if Radius includes more than One Increment.
Other Alternatives

Alternative 2– a

- Applicable if Radius includes more than one increment.

Alternative 2– b

- Fits straight cracks and small angle increments: Quadrilateral.
Other Alternatives

Alternative 2– a
- Applicable if Radius includes more than One Increment.

Alternative 2– b
- Fits Straight Cracks and Small Angle Increments: Quadrilateral.
Other Alternatives

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Alternative 2– c
- Mid Angle of Orthogonal Limiter Level Sets.
Other Alternatives

Alternative 2– a
- Applicable if Radius includes more than One Increment.

Alternative 2– b
- Fits Straight Cracks and Small Angle Increments: Quadrilateral.

Alternative 2– c
- Mid Angle of Orthogonal Limiter Level Sets.
Signed Distance

Alternative 1

- Signed Distance is Tangent to the Crack at the Tip.
Signed Distance

Alternative 1
- Signed Distance is Tangent to the Crack at the Tip.

Alternative 2–b
- Signed Distance is Tangent to the Crack Path.
**Signed Distance**

**Alternative 1**
- Signed Distance is Tangent to the Crack at the Tip.

**Alternative 2–b**
- Signed Distance is Tangent to the Crack Path.
Derivatives of Level Sets

Derivatives of Level Sets at the Integration points are Required.

- **Option 1:**
  1. Find the Signed Distance at the Integration Points.
  2. Not easy to evaluate the Derivatives.

- **Option 2:**
  1. Find the Signed Distance at the Nodes.
  2. Evaluate the Derivatives by using the Shape Functions.

- Comparing both Options for $\nabla_x \phi$: 
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- Comparing both Options for \( \nabla_x \phi \):
Alternative 0

- Enrichment does not conform to the Discontinuity.
Alternative 0

- Enrichment does not conform to the Discontinuity.

Alternative 1

- Enrichment conforms to the Discontinuity under some Restrictions away from the Tip.
**Radius Enrichment**

**Alternative 0**
- Enrichment does not conform to the Discontinuity.

**Alternative 1**
- Enrichment conforms to the Discontinuity under some Restrictions away from the Tip.

**Alternative 2–b**
- Enrichment conforms to the Discontinuity.
Stress Intensity Factors

- Inclined Center Crack: $\beta = 40^\circ$.

- SIFs Match.
Stress Intensity Factors

- Inclined Center Crack: $\beta = 40^\circ$.

![Diagram showing crack and stress intensity factors](image)

- SIFs Match.

![Graph showing stress intensity factor $K_I$](image)
Stress Intensity Factors

- Curved Center Crack: $\beta = 30^\circ$

- SIFs Convergence.
Stress Intensity Factors

- Curved Center Crack: $\beta = 30^\circ$

- SIFs Convergence.
Conclusions

- New Bases System \((\phi, \gamma)\): Triangles and a Quadrilateral.
- Improved Definition of \(\gamma\).
- Improved Stress Intensity Factors Convergence Rates.
- Improved Evaluation of Enrichment Functions.
- Flexibility and Multiplicity of Alternatives.
THANK YOU FOR YOUR ATTENTION!