Hybrid Explicit –Implicit Description of Cracks in XFEM

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Outline

Motivation

Hybrid Explicit–Implicit description of cracks
  Explicit description of the crack
  Explicit–Implicit description
  Explicit/Geometric update
Study
Summary

Conclusions and Outlook
eXtended Finite Element formulation for cracks

\[ u(x) = \sum_{i \in I} N_i(x) u_i + \sum_{j \in I_{cut}} N_j^*(x) \cdot H(x) a_j + \sum_{k \in I_{branch}} N_k^*(x) \cdot \left( \sum_{m=1}^{4} B^m b^m_k \right) \]

Continuous

Discontinuous
eXtended Finite Element formulation for cracks

\[ u(x) = \sum_{i \in I} N_i(x)u_i + \sum_{j \in I_{\text{cut}}} N_j^*(x) \cdot H(x) a_j + \sum_{k \in I_{\text{branch}}} N_k^*(x) \cdot \left( \sum_{m=1}^{4} B_m b^m \right) \]

Continuous

Discontinuous

- **Branch enrichment functions:** \( B = f(r, \theta) \).

- **Stress Intensity Factors’ evaluation:**
  Interaction integral: add the XFEM fields and auxiliary fields from pure modes of fracture—\textit{AuxiliaryFields} = \( g(r, \theta) \).
Background & Goals: XFEM for crack modeling

Crack surface can be described explicitly or implicitly.
**Background & Goals** : XFEM for crack modeling

Crack surface can be described explicitly or implicitly.

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Explicit description of the crack

- Polyline crack representation constructed by segments.
- Independent of the main mesh.
Explicit–Implicit description

- Build level sets based on the explicit crack.
Explicit–Implicit description

- Build level sets based on the explicit crack.
- Treat each segment separately.

\[ \phi_1 = \begin{cases} 
\text{shortest distance to the segment} & \text{in the segment’s domain} \\
\text{shortest distance to the tips} & \text{outside the segment’s domain} 
\end{cases} \]

- \( \phi_1 \) : signed distance.
Explicit–Implicit description

- Build level sets based on the explicit crack.
- $\phi_1$: signed distance.
- $\phi_2$: distance to the last segment tip.
Explicit–Implicit description

- Build level sets based on the explicit crack.
- $\phi_1$: signed distance.
- $\phi_2$: distance to the last segment tip.
- $\|\phi_1\| = \|\phi_2\|$: away from the crack.
Explicit–Implicit description

- Evaluation of the polar coordinates \((r, \theta)\).
Explicit–Implicit description

- Evaluation of the polar coordinates \((r, \theta)\).
- \(r = \phi_2\).
Explicit–Implicit description

- Evaluation of the polar coordinates \((r, \theta)\).
- \(r = \phi_2\).
- \(\phi_{1}^{ext} = \phi_1\) before the tip.
Explicit–Implicit description

- Evaluation of the polar coordinates \((r, \theta)\).
- \(r = \phi_2\).
- \(\phi_1^{ext} = \phi_1\) before the tip.
- \(\phi_1^{ext}\): shortest distance to the extension of last segment.
Explicit–Implicit description

- Evaluation of the polar coordinates \((r, \theta)\).

- \(r = \phi_2\).

- \(\theta = \begin{cases} 
\pi - \sin^{-1} \frac{\phi_{\text{ext}}}{\phi_2} & \text{before the tip} \\
\sin^{-1} \frac{\phi_{\text{ext}}}{\phi_2} & \text{after the tip}
\end{cases}\)
Explicit/Geometric update of the crack

- Old line mesh.
Explicit/Geometric update of the crack

- Old line mesh.
- Propagation angle from the SIFs.
Explicit/Geometric update of the crack

- Old line mesh.
- Propagation angle from the SIFs.
- Add segment/increment to old line mesh.
Crack under Bending Areais & Belytschko [2005]

- Asymmetric bending.
Summary: 2D

Implicit crack description
- Crack described implicitly by two level sets.

Hybrid explicit –implicit crack description
- Crack described explicitly independent of the mesh.
Summary: 2D

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- Crack described explicitly independent of the mesh.
- Included in the domain by level sets.
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**Implicit crack description**

- Crack described implicitly by two level sets.

**Hybrid explicit–implicit crack description**

- Crack described explicitly independent of the mesh.
- Included in the domain by level sets.
- $\phi_1$ and $\phi_2$ to locate enriched nodes.
Summary: 2D

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- Crack described explicitly independent of the mesh.
- Included in the domain by level sets.
- $\phi_1$ and $\phi_2$ to locate enriched nodes.
- $\phi_{1}^{ext}$ and $\phi_2$ to evaluate $(r, \theta)$. 
Summary: 2D – Update crack description in propagation

Implicit crack description

- Crack described implicitly by two level sets.
- Geometric update of level sets.

Hybrid explicit – implicit crack description

- Crack described explicitly independent of the mesh.
- Included in the domain by level sets.
- $\phi_1$ and $\phi_2$ to locate enriched nodes.
- $\phi_{1}^{ext}$ and $\phi_2$ to evaluate $(r, \theta)$. 
Summary: 2D – Update crack description in propagation

**Implicit crack description**

- Crack described implicitly by two level sets.
- Geometric update of level sets.
- Level set update using transport equations.

**Hybrid explicit – implicit crack description**

- Crack described explicitly independent of the mesh.
- Included in the domain by level sets.
- $\phi_1$ and $\phi_2$ to locate enriched nodes.
- $\phi_{1}^{ext}$ and $\phi_2$ to evaluate $(r, \theta)$. 
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Implicit crack description

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Hybrid explicit–implicit crack description

- Crack described explicitly independent of the mesh.
- Included in the domain by level sets.
- $\phi_1$ and $\phi_2$ to locate enriched nodes.
- $\phi_{1}^{ext}$ and $\phi_2$ to evaluate $(r, \theta)$.
- Geometric update of the crack.
3D
Explicit description of the crack

- Planar straight crack.
Explicit description of the crack

- Planar straight crack.
- Surface mesh crack representation constructed by triangles.
- Independent of the main mesh.
Explicit description of the crack

- Planar straight crack.
- Surface mesh crack representation constructed by triangles.
- Independent of the main mesh.
- Flexible mesh depending of purpose.
Explicit description of the crack

- Complex crack representation.
- Non-planar crack.
Explicit description of the crack

- Complex crack representation.
- Non-planar crack.
- Surface mesh crack representation constructed by triangles.
- Flexible mesh depending of purpose.
Explicit–Implicit description

- Build level sets based on the explicit crack.
Explicit–Implicit description

- Build level sets based on the explicit crack.
- Treat each triangle/patch separately.

\[ \phi_1 = \begin{cases} 
\text{shortest distance} & \text{inside the domain of the patch.} \\
\text{shortest distance to the edges} & \text{outside the domain of the patch.} 
\end{cases} \]

- \( \phi_1 \): signed distance.
Explicit–Implicit description

- Build level sets based on the explicit crack.
- $\phi_1$: signed distance.
- Treat each front segment separately.
- $\phi_2$: distance to the crack front segment.
Explicit–Implicit description

- Build level sets based on the explicit crack.
- $\phi_1$: signed distance.
- $\phi_2$: distance to the crack front segment.
Explicit–Implicit description

• Build level sets based on the explicit crack.
• $\phi_1$: signed distance.
• $\phi_2$: distance to the crack front segment.
• $\|\phi_1\| = \|\phi_2\|$: away from the crack.
Explicit–Implicit description

- Evaluation of the polar coordinates \((r, \theta)\).
Explicit–Implicit description

- Evaluation of the polar coordinates \((r, \theta)\).
- \(r = \phi_2\).
Explicit–Implicit description

- Evaluation of the polar coordinates \((r, \theta)\).
- \(r = \phi_2\).
- Virtually extend my crack till the end of the domain.
Explicit–Implicit description

- Evaluation of the polar coordinates \((r, \theta)\).
- \(r = \phi_2\).
- Virtually extend my crack till the end of the domain.
- \(\phi_1^{ext} = \phi_1\) before the crack front.
- \(\phi_1^{ext}\) : shortest distance to the extension of front patches.
Explicit–Implicit description

- Evaluation of the polar coordinates \((r, \theta)\).

- \(r = \phi_2\).

- \(\theta = \begin{cases} \pi - \sin^{-1} \frac{\phi_{\text{ext}}}{\phi_2} & \text{before the front} \\ \sin^{-1} \frac{\phi_{\text{ext}}}{\phi_2} & \text{after the front} \end{cases}\)
Explicit/Geometric update of the crack

- Old triangular mesh.
Explicit/Geometric update of the crack

- Old triangular mesh.
- Propagation angles from the SIFs at the front.
Explicit/Geometric update of the crack

- Old triangular mesh.
- Propagation angles from the SIFs at the front.
- Extend tips.
Explicit/Geometric update of the crack

- Old triangular mesh.
- Propagation angles from the SIFs at the front.
- Extend tips.
- Add triangles to old triangular mesh.
Crack under Bending

- Symmetric bending.
Summary: 3D

**Implicit crack description**

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**Hybrid explicit – implicit crack description**

- Crack described explicitly independent of the mesh.
Summary: 3D

Implicit crack description

- Crack described implicitly by two level sets.

Hybrid explicit –implicit crack description

- Crack described explicitly independent of the mesh.
Summary: 3D

Implicit crack description

• Crack described implicitly by two level sets.

Hybrid explicit –implicit crack description

• Crack described explicitly independent of the mesh.
Summary: 3D

Implicit crack description

- Crack described implicitly by two level sets.

Hybrid explicit –implicit crack description

- Crack described explicitly independent of the mesh.
- Convenient triangular mesh.
Summary: 3D

**Implicit crack description**

- Crack described implicitly by two level sets.

**Hybrid explicit –implicit crack description**

- Crack described explicitly independent of the mesh.
- Convenient triangular mesh.
- Included in the domain by level sets.
Summary: 3D

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- Crack described explicitly independent of the mesh.
- Convenient triangular mesh.
- Included in the domain by level sets.
- \( \phi_1 \) and \( \phi_2 \) to locate enriched nodes.
- \( \phi_1^{ext} \) and \( \phi_2 \) to evaluate \((r, \theta)\).
Summary: 3D – Update crack description in propagation

Implicit crack description

- Crack described implicitly by two level sets.
- Level set update using transport equations.

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- Crack described explicitly independent of the mesh.
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- Crack described explicitly independent of the mesh.
- Convenient triangular mesh.
- Included in the domain by level sets.
  - $\phi_1$ and $\phi_2$ to locate enriched nodes.
  - $\phi_{1,ext}$ and $\phi_2$ to evaluate $(r, \theta)$.
- Geometric update of the crack.
Conclusions

• Explicit/Geometric crack description.
• Crack is independent of the main mesh.
• Crack can be meshed flexibly based on shape.
• Hybrid explicit–implicit crack description by building level sets.
• New definition of level set functions.
• Avoid transport equations.
• Flexibility in propagation.

Outlook

• Robust scheme for 3D crack propagation.
THANK YOU FOR YOUR ATTENTION!