Material forces for 3D crack propagation in XFEM

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Outline

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II. Hybrid explicit implicit crack description.
III. Material forces as propagation criteria in FEM.
IV. Material forces as propagation criteria in XFEM.
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Traditional Crack Propagation in XFEM

\[ u(x) = \sum_{i \in I} N_i(x)u_i + \sum_{j \in I^{cut}} N_j^*(x) \cdot S(x)a_j + \sum_{k \in I^{branch}} N_k^*(x) \cdot \left( \sum_{m=1}^{4} B^m b_k^m \right) \]

- **Crack description**: level set functions.
- **Stress Intensity Factors’ evaluation**: propagation angle is determined.
- **Crack growth**: transporting LSFs.
Hybrid explicit-implicit crack description

Explicit

Geometric Crack

Implicit

Nodes for enrichment

Polar coordinates + Integration

A coordinate system \((da, db, dc)\) is set at every node
Propagation in Explicit-Implicit XFEM

1. A coordinate system is defined at each tip.

2. Crack increments are imposed at the old tips.

3. The band in between old & new tips is meshed then merged to the old mesh.
Material Forces

- Global dissipation is positive

\[ D = -\int_{\Omega} \sigma : \nabla \dot{u} \, dV - \int_{\Omega} (W \cdot I - (\nabla^T u)\sigma) : \nabla \dot{a} \, dV \geq 0 \]

- A duality between the stress driving the change in the displacement and a stress like quantity driving the crack evolution \( \dot{a} \).

- This stress like tensor is the \textbf{Eshelby} tensor or the material stress tensor as it drives the increment of the crack.

\[ \Sigma = W \cdot I - (\nabla^T u)\sigma \]
Material Forces- FEM

• In a \textit{Finite Element} setting, \( \delta u = \sum_I N^I \delta u^I \) define the displacement fields and \( \delta a = \sum_I N^I \delta a^I \) the increments caused by the material forces within the domain \( V \).

\[
- \sum_i \delta u^I \int_V (\sigma_{ij} N^I_{,j}) dV = 0; \\
- \sum_i \delta a^I \int_V (\Sigma_{ij} N^I_{,j}) dV = 0.
\]

Governing Equations

• The nodal material force \( F^I_e \) for an element \( (e) \) is evaluated by numerical integration over the element volume \( V^e \).

\[
F^I_e = -\int_{V^e} (\Sigma_{ij} N^I_{,j}) dV^e
\]
Material Forces- FEM

• The resulting total material force at a node $I$ is the sum of all nodal forces from surrounding elements.

$$F^I = \sum_{e=1}^{n_e} F^I_e$$

In the FEM, the greatest nodal material force is spotted at the crack tip node.
Material Forces- XFEM

• In the XFEM, material forces face that the crack tip does not coincide with a node but there exist distributed material forces on the nodes around the crack tip.

• A vector material force $F_{\text{tip}}$ is evaluated by summing the nodal forces within a domain $G$ surrounding the tip.

\[ F_{\text{tip}} = \sum_{\text{nodes} \subset G} F^k \]
Material Forces- XFEM- 2D

• In 2D, the domain $G$ follows a contour approach or domain approach.

- Contour approach.
- Domain approach.

Both approaches render similar tip force
Material Forces- XFEM- 3D

- The domain $G$ with a radius $R_G$ and width $W_G$:

1. For a frontal point with coordinates $\{x_{tip}, y_{tip}, z_{tip}\}$, the set of elements inside a tube are assigned whereby their nodes satisfy:

$$I_{tube} \subset \{r - R_G \leq 0\}$$
2. From the coordinate system \((da, db, dc)\) at each node, the tangent values with respect to coordinates \((x, y, z)\) of nodes \(I_{tube}\) are evaluated

\[
dc_x = (\nabla_x c).(x - x_{tip}),
\]
\[
dc_y = (\nabla_y c).(y - y_{tip}),
\]
\[
dc_z = (\nabla_z c).(z - z_{tip}).
\]

The magnitude \(C_G = \sqrt{dc_x^2 + dc_y^2 + dc_z^2}\) is then defined. The elements that denote the material forces' domain \(G\) have the nodes satisfying:

\[
I^G \subset \{C_G - w_G \leq 0\}
\]
Parameters & limitations

- Larger $R_G$ renders better results to a certain limit!

- $\mathcal{W}_G$ extends to the whole crack front.

- Varying $\mathcal{W}_G$ at every individual point.

- Equal $\mathcal{W}_G$ for every individual point.
Parameters & limitations

- Mesh dependency

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Material Forces based Propagation

- In 3D, to account for varying increments at the front of the geometric crack, the increment evolution in a discrete setting $\dot{a} = \Delta a$ at a frontal point "i" is related to the prescribed increment $V$ by:

$$\Delta a_i = V \cdot \vec{n}_i$$

with

$$\vec{n}_i = \frac{F_i}{\max \{|| F_1 ||, ..., || F_{ntips} ||\}}$$

- In 2D, the crack increment is now reduced to:

$$\Delta a_{tip} = V \cdot \vec{n}_{tip}$$
Numerical Results

• Asymmetric bending test: \( n^0 \text{Elements} = 3527; \)
  \[ V = 7; \quad k = 0; \]
  \[ w_G = 5; \quad R_G = 11; \]
Numerical Results

- Asymmetric bending test: \( n^0 \text{Elements} = 3527; \)
  \[
  V = 7; \quad k = \infty; \\
  w_G = 5; \quad R_G = 13;
  \]
Numerical Results

- Torsion test case: \( \text{n}^\circ \text{Elements} = 2351; V = 5; \)
  \( w_G = 7; \quad R_G = 13; \)
Numerical Results

• Domain approach & averaging
Conclusions

• **Material forces** as geometric propagation criteria in 3D XFEM.
• Effects of the width $\mathcal{W}_G$ and radius $R_G$ of the domain.
• Having an equal domain is very hard: Averaging is required.
• Limited dependency on the mesh size.
• Similar crack paths to existing propagation results.

Outlook

• Further Studies on $\mathcal{W}_G$.
• Comparative study between material forces and **MCSC** in XFEM.