SIMULATION OF CRACKS WITH XFEM AND HANGING NODES

A. ALIZADA∗, T.-P. FRIES†

∗Chair for Computational Analysis of Technical Systems (CATS)
RWTH Aachen University, Schinkelstr. 2, 52062 Aachen, Germany
e-mail: alizada@cats.rwth-aachen.de, web page: http://www.xfem.rwth-aachen.de

†Chair for Computational Analysis of Technical Systems (CATS)
RWTH Aachen University, Schinkelstr. 2, 52062 Aachen, Germany
e-mail: fries@cats.rwth-aachen.de, web page: http://www.xfem.rwth-aachen.de

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Summary. In this paper, XFEM approximations are applied on meshes with hanging nodes for crack simulations. Thereby, the step-enrichment is used along the crack path and an adaptive refinement is used near the crack-tip. No crack-tip enrichments are thus required.

1 INTRODUCTION

The extended finite element method (XFEM) ¹ is a frequently used enriched finite element technique for the approximation of solutions with discontinuities, singularities, or other locally non-polynomial phenomena. The method enables the inclusion of known solution properties into the approximation space. Thereby, optimal convergence rates are achieved for solutions that involve kinks, jumps, and also high gradients ² within elements. The simulation is typically carried out on simple (often regular) meshes without mesh alignment or refinement near the discontinuities.

In crack problems, a jump in the displacement field appears across the crack interface. Moreover, at the crack-tip a singularity or general high gradient is present in the stress and strain fields. In this work, the step enrichment with the Heaviside function is used along the crack path as in common XFEM simulations. However, at the crack-tip rather than an enrichment, a mesh refinement is realized based on 'hanging node' elements. Thereby, the knowledge of the analytical behavior of the solution at the crack-tip is no longer needed. This is important for material and fracture models with unknown analytical solutions at the crack-tip. In the 'hanging node' elements special shape functions with partition of unity property are used.

2 Refinement algorithm

Uniform meshes are often not computationally appropriate to solve systems of partial differential equations, where singularities, gradients, or discontinuities need to be captured. Using coarse elements in the vicinity of such regions leads to bad accuracy, and using small elements in the whole domain is computationally not efficient. The adaptive refinement method allowing hanging nodes adjusts the mesh resolution for better accuracy only where it is required and hereby reduces the number of unknowns drastically, compared to global refinement.
In this paper, a uniform mesh with only quadrilateral elements is taken as an initial mesh. For the elements near a crack-tip an adaptive mesh refinement is applied. The following properties are found to be useful and the mesh is called admissible if:

- Two elements are called neighbour elements, if they have one common edge and 2 common nodes.
- The nodes are called regular if they are vertex nodes of the element. If the node lies on the edge and is not regular, then it is a hanging node.
- New nodes generated through the refinement are always located in the middle of the edge.
- There is never more than one refinement step difference between any two neighbouring elements. It follows that only one hanging node per element edge is allowed.
- Each refined element (parent) contains always four subelements (children).

It is important to mention that through the necessary refinement of elements near crack-tips some edges of an element can have more than one hanging node in a preliminary refinement step. Since this is not admissible for our mesh, the appropriate neighbour elements also have to be refined. It can be shown that at most 2 neighbour elements to the element with the crack-tip have to be refined. In Fig. 1, the allowed types of elements in our mesh are shown. As an example, Fig. 2 shows an admissible mesh.

![Figure 1: The allowed types of elements in the proposed mesh: (a) an element has no hanging nodes, (b) an element has one hanging node, (c) and (d) an element has two hanging nodes, (e) an element has three hanging nodes, (f) an element has four hanging nodes.](image)

3 **XFEM for cracks**

In this work, the standard shifted XFEM formulation for enriched elements is used:

\[
u^h (\mathbf{x}) = \sum_{i \in I} N_i (\mathbf{x}) u_i + \sum_{i \in I^*} N_i (\mathbf{x}) (\psi (\mathbf{x}) - \psi (\mathbf{x}_i)) a_i.
\]

- **strd. FEM approx.**
- **enrichment part**
The XFEM approximation of function $u^h(x)$ consists of a standard FEM approximation part plus the enrichment part. Special FE shape functions are used in the elements with hanging nodes\textsuperscript{5}. These shape functions build a partition of unity no matter how many hanging nodes are present in an element.

This is crucial for the enrichment which relies on the partition of unity concept. The enrichment part consists of the standard FEM shape functions $N_i(x)$, the global enrichment function $\psi(x)$ and the unknowns $a_i$, which are defined at nodes in $I^{\star} \subset I$. By means of $\psi(x)$, special solution characteristics can be introduced into the approximation space. In this paper, the step enrichment with the Heaviside function $^3\!_1$ along the crack path is applied.

$$\psi(x) = H(\phi(x)) = \begin{cases} 
0 & : \phi(x) \leq 0, \\
1 & : \phi(x) > 0,
\end{cases}$$

where $\phi(x)$ is the level-set function. We use the shifted XFEM approximation, therefore, the step enrichment function vanishes in the blending elements $^4$.

For the crack-tip rather than an enrichment, the adaptive mesh refinement is realized. That means, the knowledge of the analytical behavior of the solution at the crack-tip is no longer needed. This is important for material and fracture models with unknown analytical solutions at the crack-tip. After the adaptive mesh refinement 'hanging nodes' are present. As an alternative to the constrained approximation and other methods special shape functions with partition of unity property are used for the 'hanging nodes'$^5$.

4 Numerical results and outlook

In order to validate the proposed idea, a standard crack problem in the frame of linear elastic fracture mechanics is considered. The results are compared with the the solution obtained by common XFEM\textsuperscript{1} with crack-tip enrichment functions. For simplicity, first a straight crack is simulated (Fig. 2). To approximate the discontinuity along the crack, the Heaviside enrichment is applied. The crack-tip element is refined to resolve the singularity. The application of the proposed idea for other material models will be done in the future.

REFERENCES


Figure 2: The crack problem considered for validation.